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A. J. Krener

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Department of Mathematics
University of California, Davis
Davis, CA 95616-8633

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14. ABSTRACT

The concept of a control bifurcation grew out of an AFOSR PRET project to study the robust stabilization of axial flow compressors. Several versions of the Moore Greitzer compressor model were used in this study. McCaughan [28] had studied the classical bifurcations that were present in the three dimensional Moore Greitzer equations(MG3) [29] as the throttle and other parameters were varied. Liaw and Abed [27] considered the throttle and other parameters as a control and developed feedback laws to change the criticality of the primary Hopf bifurcation in MG4.

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Final Report:
Bifurcations of Control Systems with Application to Flutter
AFOSR F49620-01-1-0202

A. J. Krener
Department of Mathematics
University of California
Davis, CA 95616-8633

Objectives

We have developed a theory of control bifurcations. Loosely speaking a control bifurcation is a loss of linear stabilizability and as such they can be critical to the success of a nonlinear control system.

Status of Effort

We have classified all control bifurcations through codimension two and have studied how they can be stabilized using truly nonlinear feedbacks.

Accomplishments/New Finding

The concept of a control bifurcation grew out an AFOSR PRET project to study the robust stabilization of axial flow compressors. Several versions of the Moore Greitzer compressor model were used in this study. McCaughan [28] had studied the classical bifurcations that were present in the three dimensional Moore Greitzer equations (MG3) [29] as the throttle and other parameters were varied. Liaw and Abed [27] considered the throttle parameter as a control and developed feedback laws to change the criticality of the primary Hopf bifurcation in MG4. We realized that there was more than classical bifurcations in these models, there was a new type of bifurcation, a control bifurcation. With this support from AFOSR we studied such bifurcations. Our efforts have stimulated other workers around the world and we now have some understanding of control bifurcations in low codimensions.

A clssical bifurcation can occur in a parameterized family of differential or difference equations as the parameter is varied. Consider the equations

$$\dot{x} \;=\; f(x,\mu)$$

where the state x is n dimensional and, for simplicity, the parameter μ is one dimensional. The equilibria x, μ are the solutions of the equation

$$0 \;=\; f(x,\mu).$$

This is $n+1$ equations in n unknowns so typically there is a one parameter family of equlibria $x_e = x_e(\mu)$.

Two equilibria are topologically equivalent if there is a local homeomorphism of the state space carrying one equlibrium and its local orbits to the other and its local orbits while preserving the time direction of orbits but not necessarily the exact time. A classical bifurcation occurs at an equilbrium which is not topologically equivalent to its neighboring equilibria.

The linear approximating system at the μ^{th} equilbrium in displacement coordinates is

$$\dot{z} \;=\; F(\mu)z$$

where $z = x - x_e(\mu)$ and

$$F(\mu) = \frac{\partial f}{\partial x}(x_e(\mu), \mu).$$

The usual way that a classical bifurcation occurs is that one or more eigenvalues of $F(\mu)$ cross the imaginary axis as μ is varied. To determine the simple ways that this can occur, one uses the concept smooth equivalence and its resulting Poincaré normal forms. Two equilibria are smoothly equivalent if there is a local diffeomorphism of the state space carrying one equlibrium and its local orbits to the other and its local orbits preserving the exact time. The normal forms are represntatives of the resulting equivalence classes. The drawback of using smooth equivalence is that there are too many equivalence classes. So one restricts one attention to smoooth equivalence of the lowest degree terms in power series expansions of the systems about teir respective equilibria. A degree d normal form of a system is a particularly simple system which is smoothly equivalent through terms of degree d. The normal form may or may not determine the topolgical type of the equilibrium. But the cases where it does not are usually of higher codimension in the class of systems. Arnold and Ilyashenko [15] have given classification of all singular equilibria in codimensions one and two. This leads to a classification of all bifurcations up to codimension two [16] by adding the parameter to the state with $\dot{\mu} = 0$.

A control system

$$\dot{x} = f(x, u),$$

where the state x is n dimensional and, for simplicity, the control u is one dimensional, does not need a parameter to bifurcate. The equilibria x, u are the solutions of the equation

$$0 = f(x, u).$$

Again this is $n+1$ equations in n unknowns so typically there is a one parameter family of equlibria $x_e = x_e(\mu)$ where μ is the set value of the control or of a state.

The most widely used equivalence of control systems is smooth feedback equivalance. Two control systems with equilibria

$$\dot{x} = f(x, u), \qquad 0 = f(x_e, u_e)$$

$$\dot{z} = g(z, v), \qquad 0 = g(x_e, v_e)$$

are smoothly feedback equivalent if there is a local diffeomorphism

$$\begin{aligned} z &= \phi(x) \\ v &= \kappa(x, u) \end{aligned}$$

between the two systems at the equilibria.

This is too fine an equivalence, there are too many equivalence classes. We might try for a coarser equivalence; two equilibria are closed loop, topologically equivalent if there are continuous feedbacks

$$\begin{aligned} u &= \kappa(x) \\ v &= \lambda(z) \end{aligned}$$

such that the closed loop systems are topologically equivalent.

But this definition is not an equivalence relation (not transitive) and it ignores the most important systems theoretic property of an equilibrium of a control system, whether it is stabilizable by state feedback. Therefore we add the requirement that the feedbacks locally asymptotically stabilize the systems.

A equilibrium of a control system is *structurally stabilizable* if it and all nearby equilibria of all nearby systems are locally asymptotically stabilizable by continuous feedbacks.

A control system is *locally parameterically stabilizable* at $x_e(\mu_c), u_e(\mu_c)$ if there exists a continuous, parameterized feedback

$$u = k(x, \mu)$$

defined for all μ near μ_c and x near $x_e(\mu_c)$ which locally, asymptotically stabilizes the system to $x_e(\mu)$.

A *control bifurcation* [8] occurs at an equilibrium x_e, u_e which is not locally parameterically stabilizable.

Perhaps this should be called "stabilizability bifurcation" but this terminology is too awkward.

In practice, control bifurcations occur when the linear part of the system loses stabilizability. But this is not always true, here is a system which loses linear stabilizability but does not experience a control bifurcation,

$$\dot{x} = -u^2 x - x^3$$

where $x \in I\!R, u \in I\!R$. This system has a locus of equilibria given by $x_e = 0, u_e = \mu$. For any $\mu \neq 0$ the equilibrium is linearly stable hence linearly stabilizable. At $\mu = 0$ the system is not linearly stabilizable but is nonlinearly stable hence nonlinearly stabilizable so there is no control bifurcation.

On the other hand, a system can experience a control bifurcation without a change in its linear stabilizability, here is an example.

$$\dot{x} = x + xu$$

where $x \in I\!R, u \in I\!R$. This system has a locus of equilibria given $x_e = 0, u_e = \mu$ and is not linearly stabilizable for any μ. For any $\mu < -1$, the equilibrium is stabilizable by the feedback $u = k(x, \mu) = \mu$. But it is not stabilizable to the equilibria where $\mu \geq -1$ so it undergoes a control bifurcation at $\mu = -1$.

If a control system is locally parameterically stabilizable then any other system that is smoothly feedback equivalent to it is also locally parameterically stabilizable. We have developed a theory of normal forms (relative to the

smooth feedback group) and they are very useful in classifying the simplest ways a control bifurcation can happen [22], [23], [24], [8]. There is a close correspondence between the simplest classical and control bifurcations. Very often when the loop is closed around a system with a control bifurcation the result is an inevitable classical bifurcation in the closed loop dynamics.

We have classified all control bifurcations in codimensions one and two, see [20], [21], [1], [2], [7], [6], [23], [24], [8], [11], [26]. The complete classification will appear in [10]

We start with systems

$$\dot{x} = f(x,u)$$

with a scalar control u. The simplest control bifurcation is the fold [8]. It is the only codimension one control bifurcation. Its normal form is

$$\begin{aligned} \dot{x}_1 &= \alpha x_1 + \gamma x_1 x_{21} + \delta x_{21}^2 + O(x,u)^3 \\ \dot{x}_2 &= A_2 x_2 + B_2 u + O(x,u)^2 \end{aligned}$$

where $x_1 \in I\!R$, $x_2 \in I\!R^{n-1}$ Without loss of generality, we can assume that A_2, B_2 are in Brunovsky form and $\alpha > 0, \gamma \geq 0, \delta \geq 0$

The equilibria of this system can be parameterized by the first component x_{21} of x_2 which we denote by μ. Then the equilibria near $\mu = 0$ are given by

$$\begin{aligned} x_1(\mu) &= -\frac{\delta}{\alpha}\mu^2 + O(\mu)^3 \\ x_{21}(\mu) &= \mu \\ x_{2i}(\mu) &= O(\mu)^2, \quad i = 2, \dots, n-1 \\ u(\mu) &= O(\mu)^2. \end{aligned}$$

The linearization around μ^{th} equilibrium in displacement coordinates

$$z = x - x(\mu) \quad v = u - u(\mu)$$

is given by

$$\begin{aligned} \dot{z} &= \left(\left[\begin{array}{c|ccc} \alpha + \gamma\mu & 2\delta\mu & 0 & \dots & 0 \\ \hline 0 & & A_2 & & \end{array} \right] + O(\mu)^2 \right) z \\ &+ \left(\left[\begin{array}{c} 0 \\ \hline B_2 \end{array} \right] + O(\mu)^2 \right) v. \end{aligned}$$

To leading order in μ this is a parameterized family of linear systems

$$\dot{z} = A(\mu)z + Bv + O(\mu)^2$$

whose controllability matrices are

$$\left[\begin{array}{c} A^{n-1}B \dots B \end{array} \right] = \left[\begin{array}{cc} 2\delta\mu & 0 \\ 0 & I \end{array} \right] + O(\mu)^2.$$

Notice that the determinant of the controllability matrix changes sign at $\mu = 0$. This is a manifestation of the control bifurcation. The effect of the integrated control action on z_1 reverses direction at $\mu = 0$.

Suppose we try to close the loop by a continuous parameterized feedback $u = \kappa(x, \mu)$ so that for each small μ, the closed loop system

$$\dot{x} = f(x, \kappa(x, \mu))$$

is asymptotically stable to $x(\mu)$.

If the feedback is smooth then in displacement coordinates it takes the form

$$v = K_1(\mu)z_1 + K_2(\mu)z_2 + \dots.$$

The linear part of the resulting closed loop system at the μ^{th} equilibrium is

$$\dot{z} = \left(\left[\begin{array}{c|c} \alpha + \gamma\mu & 2\delta\mu \quad 0 \quad \dots \quad 0 \\ \hline B_2 K_1 & A_2 + B_2 K_2 \end{array} \right] + O(\mu)^2 \right) z$$

and so the closed loop system undergoes a classical fold bifurcation near $\mu = 0$.

The next simplest bifurcation is of codimension two and is called the transcritical control bifurcation [8]. It is also called the transcontrollable bifurcation. Its normal form is

$$\begin{aligned} \dot{x}_1 &= \beta x_1^2 + \gamma x_1 x_{21} + \delta x_{21}^2 + O(x, u)^3 \\ \dot{x}_2 &= A_2 x_2 + B_2 u + O(x, u)^2 \end{aligned}$$

where again $x_1, u \in I\!R$, $x_2 \in I\!R^{n-1}$ and A_2, B_2 are in Brunovsky form. It can be seen as a degenerate form of the fold control bifurcation where $\alpha = 0$. When $\alpha = 0$ there is an extra term in the normal form, βx_1^2 that cannot be eliminated by the feedback group.

The generic case is when the quadratic form $\beta x_1^2 + \gamma x_1 x_{21} + \delta x_{21}^2$ is nondegenerate. If this form is sign definite then $x_1 = 0, x_2 = 0, u = 0$ is an isolated equilibrium. If it takes on both positive and negative values then there are two curves of equilibria crossing at $x_1 = 0, x_2 = 0, u = 0$. Each of these curves can be parameterized by $\mu = x_{21}$. The linearization about any of these equilibria is linearly controllable except at $\mu = 0$ where the curves cross. The controllability matrix changes sign at $\mu = 0$ on each branch of equilibria which is the reason that this is called a transcontrollable bifurcation. If we again try to find a parameterized feedback $u = \kappa(x, \mu)$ so that for each small μ, the closed loop system

$$\dot{x} = f(x, \kappa(x, \mu))$$

is asymptotically stable to the equilbria on one branch then typically there is a classical transcritical bifurcation in the closed loop system.

We have also discovered nonlinear ways of stabilizing these bifurcations where linear methods fail. Both the fold and transcontrollable bifurcations can be put in the normal form

$$\begin{aligned} \dot{x}_1 &= \alpha x_1 + \beta x_1^2 + \gamma x_1 x_{21} + \delta x_{21}^2 + O(x, u)^3 \\ \dot{x}_2 &= A_2 x_2 + B_2 u + O(x, u)^2 \end{aligned}$$

In either case there is an exchange of controllability at $\mu = 0$ so we choose the piecewise linear feedback

$$u = K_1 x_1 + K_2 x_2$$

where $K_1 = K_1^{\pm}$ as $\pm x_1 > 0$ and $A_2 + B_2 K_2$ is Hurwitz. Then the x_2 coordinate is stable in the first approximation so we need only worry about the stability of x_1. If $\alpha < 0$ the there is no control bifurcation as x_1 is also stable in the first approximation and there is nothing more to do, we can set $K_1^{\pm} = 0$.

If $\alpha \geq 0$ then there is a control bifurcation and we must choose $K_1^{\pm}$ appropriately. The reduction of the problem of stabilizing the overall system to that of stabilizing a small number of coordinates, in this case one coordinate, is reminscent of how the center manifold theorem is used to reduce the study of the stability of an equilibrium of a higher dimensional ODE to the study of the stability of the dynamics restricted to the center manifold. With support from this grant we have developed the controlled center dynamics technique for stabilizing the linearly unstabilizble part of the system.

Following [6] and [?], we seek a piecewise smooth, approximately invariant manifold of the form

$$x_2 = L^{\pm} x_1 + O(x_1)^2 = \begin{bmatrix} L_1^{\pm} \\ \vdots \\ L_{n-1}^{\pm} \end{bmatrix} x_1 + O(x_1)^2$$

for the closed loop dynamics. The approximately invariant manifold assumption is that

$$\frac{d}{dt} x_2 = L^{\pm} \frac{d}{dt} x_1 + O(x)^2$$

or equivalently

$$(A_2 + B_2 K_2)\, L^{\pm} x_1 + B_2 K_1^{\pm} x_1 = \alpha x_1.$$

This reduces to

$$K_1^{\pm} = L_1^{\pm} p_2(\alpha), \quad L_i^{\pm} = L_1^{\pm} \alpha^{i-1}, \quad i = 2, \ldots, n-1$$

where $p_2(s)$ is the characteristic polynomial of the Hurwitz matrix $A_2 + B_2 K_2$. Since $\alpha \geq 0$, $p_2(\alpha) \neq 0$ so we can parameterize the first part of the feedback by $L_1^{\pm}$ instead of $K_1^{\pm}$.

The dynamics on this manifold is

$$\dot{x}_1 = \alpha x_1 + \left(\beta + \gamma L_1^{\pm} + \delta (L_1^{\pm})^2\right) x_1^2 + O(x)^3$$

First we consider the transcontrollable bifurcation where $\alpha = 0$. By assumption the quadratic $\beta + \gamma L_1^{\pm} + \delta (L_1^{\pm})^2$ takes on both positive and negative values as $L_1^{\pm}$ varies. So we choose $L_1^{\pm}$ so that

$$\pm \left(\beta + \gamma L_1^{\pm} + \delta (L_1^{\pm})^2\right) < 0$$

and we have locally stabilized x_1. The stability of the overall system follows from Lyapunov arguments.

For a fold control bifurcation where $\alpha > 0$, we certainly cannot achieve asymptotic stability. But if α is small enough we can achieve practical stability. Without loss of generality, $\beta = 0$. We choose small L_1^+ so that $\alpha x_1 + (\gamma L_1^+ + \delta (L_1^+)^2)\, x_1^2$ has a small positive root x_1^+. Then this is a locally stable equilbrium. Similarly we choose small L_1^- so that $\alpha x_1 + (\gamma L_1^- + \delta (L_1^-)^2)\, x_1^2$ has a small negative root x_1^-. This is also a locally stable equilbrium and so locally x_1 goes to either $x_1^\pm$. This is called practical stabilizability as we can choose $L_1^\pm$ to make these two stable equilibria $x_1^\pm$ close to $x_1 = 0$. We have not stabilized to $x_1 = 0$, this is impossible as $\alpha > 0$, but we have stabilized to a small neighborhood of it, $[x_1^-, x_1^+]$. Again the practical stability of the overall system follows from Lyapunov arguments [6].

The above analysis analogous to the stability analysis of an equilibrium of an ODE using the center manifold theorem. The method of the controlled center dynmaics is a way for stabilizing (or practically stabilizing) a control system that are not linearly stabilizable. First one uses linear feedback to stabilize the linearly controllable modes and then one uses invariant manifold techniques to study the stabilizability of the reduced system consisting of the linearly unstable and uncontrollable modes.

The fold is the only codimension one control bifurcation of a scalar input system. The transcontrollable bifurcation is of codimension two. The other control bifurcations of codimension two are as follows.

For a scalar input system there are two and they both involve a system with two linearly unstabilizable modes at some critical equilibrium. They differ in the linear part of the unstabilizable modes. The first has two real, distinct positive eigenvaues and second has a complex conjugate pair of eigenvalues with positive real part. The latter is sometimes called the Hopf control bifurcation [8] but perhaps this term should be reserved for the codimension three case when the real parts of the eigenvalues are zero.

Other higher codimension control bifurcations of scalar input systems are where one or both of the real uncontrollable eigenvalues are zero and where the eigenvalues of the uncontrollable linear part are the same and its Jordan form is not diagonal. Even higher codimension cases include the case where the eigenvalues of the uncontrollable linear part are the same and its Jordan form is diagonal and where the eigenvalues are zero.

There is one codimension two control bifurcation of multi-input systems. It is a double fold.

If there is more than one input then a control bifurcation usually takes place at an equilibrium where the controllablity (Kronecker) indices change. For example if the dimension of the state is $n = 2k$ and the dimension of the control is $m = 2$ then generically the controllability indices are $\{k, k\}$. A codimension one control bifurcation takes place when the controllability indices become $\{k+1, k-1\}$.

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Personnel Supported

Arthur J. Krener, Principle Investigator and Professor

Issa Tall, Postdoctoral Research Associate

Boumediene Hamzi, Postdoctoral Research Associate

Carmeliza Navasca, Graduate Student

Randy Robertson, Graduate Student

Publications

See [1]-[13] of Bibliography above.

Interactions/Transitions

Presentations of A. J. Krener

University of Wuerzburg. 3/29/01, Nonlinear Observer Design in the Siegel Domain.

Nonlinear Control Network Workshop, Irsee, Germany, 4/1/01, Control Bifurcations.

London Mathematical Society Workshop, Bath, England, 6/19/01, Nonlinear Observer Design in the Siegel Domain.

Nonlinear Control Design Symposium (NOLCOS 2001), St. Petersburg, Russia, 7/4/01, Nonlinear Observer Design in the Siegel Domain"

Nonlinear Control Design Symposium (NOLCOS 2001), St. Petersburg, Russia, 7/05/02 Control Bifurcations

SIAM Meeting on Control and its Applications, San Diego, 7/27/01, Control Bifurcations

Colloquium, UCSB, 7/26/01, Nonlinear Observer Design in the Siegel Domain

Red Raider Symposium, Texas Tech, 11/9/01, Nonlinear Observers

Oberwolfach Workshop on Control Theory, 3/1/02, Nonlinear Observers

Conference on Future Directions in Control, Washington, DC, 4/25/02, The Interaction between Control and Dynamical Systems.

American Control Conference, 5/7/02, Observers for Linearly Unobservable Nonlinear Systems.

Bedlow Workshop, Poland, 6/18/03, The Local Convergence of the Extended Kalman Filter and the Global Convergence of the Minimum Energy Estimator.

U. Potsdam, Germany, 6/13/03, The Local Convergence of the Extended Kalman Filter and the Global Convergence of the Minimum Energy Estimator.

Weierstrass Institute, Berlin, Germany, 6/12/03,Control Bifurcations.

Lund Institute of Technology, Sweden, 5/22/03, Control Bifurcations.

Mittag Lefler Institute, Stockholm 5/15/03, The Local Convergence of the Extended Kalman Filter and the Global Convergence of the Minimum Energy Estimator.

Mittag Lefler Institute, Stockholm, 4/3/03, Control Bifurcations.

MAE Seminar, UCD, 1/30/03, The Local Convergence of the Extended Kalman Filter and the Global Convergence of the Minimum Energy Estimator.

ECE Control Seminar, UCD. 1/15/03, Control Bifurcations.

Conference on New Directtions in Mathematical Systems Theory and Optimiztion, 11/16/02, KTH, Stockholm, The Local Convergence of the Extended Kalman Filter.

Symposium on New Trends in Nonlinear Dynamics and Control, and Their Applications, 10/18/02, Naval Postgradutae School, Monterey, CA,, The Global Convergence of the Minimum Energy Estimator.

Colloquium, Naval Postgraduate School, 11/20/03, The Local Convergence of the Extended Kalman Filter and the Global Convergence of the Minimum Energy Estimator.

Workshop at Texas Tech, 11/15/03, Control Bifurcations in Low Codimensions.

AFOSR Contractors Meeting, Destin, FL, 9/10/03, The Local Convergence of the Extended Kalman Filter and the Global Convergence of the Minimum Energy Estimator.

Workshop on Invariant Manifolds and Model Reduction, ETH, Zurich, Switzerland, 8/23/03, Central Controlled Dynamics.

Colloquium, UCSB, 7/28/03, Control Bifurcations in Low Codimensions.

Presentations of B. Hamzi

Symposium on "New Trends in Nonlinear Dynamics and Control and their Applications", Naval Postgraduate School, Monterey. October 2002, "Practical Stabilization of Systems with a Fold Control Bifurcation".

41nd IEEE Conference on Decision and Control, Las-Vegas. December 2002, "Analysis and Stabilization of nonlinear systems with a Zero-Hopf control bifurcation".

41nd IEEE Conference on Decision and Control, Las-Vegas. December 2002, "Quadratic stabilization of systems with period doubling bifurcation".

The Sixth Southern California Nonlinear Control's Workshop, U.C. San Diego. May 2003, "Practical Stabilization of Systems with a Fold Control Bifurcation".

Symposium on "Computation, Control and Biological Systems", Montana State University, Bozeman, Montana. July 2003, "Piecewize linear stabilization of systems with control bifurcations".

42nd IEEE Conference on Decision and Control, Hawaii. December 2003, "Control of Center Manifolds".

New Discoveries

None.

Honors/Awards

Arthur J. Krener was named a Fellow of IEEE in 1990.

Arthur J. Krener received a John Simon Guggenheim Fellowship in 2001.

Arthur J. Krener received the W. T. and Idalia Reid Prize from SIAM in 2004.

Carmeliza Navasca tied for top honors in the Best Student Poster Prize at the ACM's First Richard A. Tapia Symposium 2001, Oct 18-20, 2001 in Houston, Texas. The title of her award winning presentation is: the Local Solution of the Dynamic Programming Equations in Discrete-Time.